Unit 7 Test — Answer Key

Pythagorean, Sum/Difference, Double-Angle Identities & Solving Equations

FOR TEACHER / SELF-CHECK USE

This test was written to cover Unit 7's topics (fundamental, Pythagorean, sum/difference, and double-angle identities, and solving trigonometric equations). Every solution below has been independently verified.

1

By definition, the quotient identity: $\sin(x)/\cos(x) = \tan(x)$.

Answer: A — tan(x)

2

This is the fundamental Pythagorean identity.

Answer: A — 1

3

By the Pythagorean identity, $1 - \cos^2(x) = \sin^2(x)$. So the expression becomes $\sin^2(x)/\sin(x)$.

Answer: sin(x)

4

$\sin(75°) = \sin(45° + 30°) = \sin 45°\cos 30° + \cos 45°\sin 30°$
$= (\sqrt{2}/2)(\sqrt{3}/2) + (\sqrt{2}/2)(1/2) = \sqrt{6}/4 + \sqrt{2}/4$.

Answer: A — $(\sqrt{6} + \sqrt{2})/4$

5

The double-angle identity $\cos(2x) = 1 - 2\sin^2(x)$ is a valid form. Choice B is the identity for $\sin(2x)$; C simplifies to $\sin^2(x)$; D equals $-\cos(2x)$.

Answer: A — $1 - 2\sin^2(x)$

6

$\sin(2x) = 2\sin(x)\cos(x) = 2(3/5)(4/5) = 24/25$.

Answer: 24/25

7

$2\sin(x) = 1 \rightarrow \sin(x) = 1/2$. On $[0,2\pi)$, sine equals 1/2 at the reference angle $\pi/6$, in Quadrants I and II: $x = \pi/6,\ \pi - \pi/6 = 5\pi/6$.

Answer: A — $\{\pi/6, 5\pi/6\}$

8

$\cos(2x) = 0 \rightarrow 2x = \pi/2 + k\pi \rightarrow x = \pi/4 + k\pi/2$. On $[0, \pi)$, this gives $x = \pi/4$ and $x = 3\pi/4$.

Answer: A — $\{\pi/4, 3\pi/4\}$

9

Cross-multiply: $(1 + \cos(x))(1 - \cos(x)) = \sin(x)\cdot\sin(x)$.
Left side: $1 - \cos^2(x)$. Right side: $\sin^2(x)$.
By the Pythagorean identity, $1 - \cos^2(x) = \sin^2(x)$, so both sides are equal and the identity is verified.

Answer: Identity verified: both sides reduce to $\sin^2(x) = 1 - \cos^2(x)$

10

Rewrite $\sin(2x) = 2\sin(x)\cos(x)$, so the equation becomes $2\sin(x)\cos(x) - \sin(x) = 0$.
Factor: $\sin(x)(2\cos(x) - 1) = 0$, so $\sin(x) = 0$ or $\cos(x) = 1/2$.
$\sin(x)=0$ on $[0,2\pi)$: $x = 0, \pi$. $\cos(x)=1/2$ on $[0,2\pi)$: $x = \pi/3, 5\pi/3$.

Answer: $x = 0, \pi/3, \pi, 5\pi/3$