Name: _____ Date: _____ Class: _____

ESSENTIAL QUESTION

How do you translate real-world periodic data — a maximum, a minimum, and a period — into a precise mathematical model of the form y = A·sin(Bx + C) + D?

Lesson Overview

A sinusoidal model has the form y = A·sin(Bx + C) + D (or with cosine). Each parameter controls a specific feature of the graph: |A| is the amplitude, period = 2π/|B|, phase shift = −C/B, and D is the vertical shift (midline). Recipe: find A and D from the max/min, find B from the period, find C from a known point.

y = A·sin(Bx + C) + D

From Data	A = (max−min)/2; D = (max+min)/2; B = 2π/period; C: use a known point
Meanings	Amplitude=|A|; Period=2π/|B|; Phase shift=−C/B; D=vertical shift (midline)
Tip	Max of cosine occurs at x=0 — if a max is given at a specific x, cosine is often cleaner.

Worked Examples

EXAMPLE 1

A city's monthly temperature has a max of 88°F in July (t=7) and a min of 28°F in January (t=1). Write T(t)

- A = (88−28)/2 = 30. D = (88+28)/2 = 58
- Period = 12 months → B = 2π/12 = π/6
- Use cosine: maximum occurs at t=7 → T(t) = 30cos(B(t−7)) + 58

Answer: T(t) = 30cos(π(t−7)/6) + 58

EXAMPLE 2

A Ferris wheel has diameter 40m, center 25m high, period 60s. Rider starts at the bottom. Write h(t)

- A = 40/2 = 20 (radius = amplitude). D = 25 (center height = midline)
- Period = 60s → B = 2π/60 = π/30
- Rider starts at the bottom (minimum) → use −cosine

Answer: h(t) = −20cos(πt/30) + 25

EXAMPLE 3

Amplitude 4, period 2π/3, phase shift π/6 right, midline y=1. Write the equation

- A = 4, D = 1
- Period = 2π/3 → B = 2π/(2π/3) = 3
- Phase shift = −C/B = π/6 → C = −π/2

Answer: y = 4sin(3x − π/2) + 1

EXAMPLE 4

y = 3sin(2x + π/4) − 1 models a wave. Identify amplitude, period, phase shift, midline

- Amplitude: |A| = |3| = 3
- Period: 2π/|B| = 2π/2 = π
- Phase shift: −C/B = −(π/4)/2 = −π/8 (shift left π/8). Midline: y = D = −1

Answer: Amplitude = 3, Period = π, Phase shift = π/8 left, Midline y = −1

EXAMPLE 5

High tide 12ft at 2AM, low tide 2ft at 8AM. Write h(t), t=hours after midnight

- A = (12−2)/2 = 5. D = (12+2)/2 = 7
- Half-period = 8−2 = 6 hours → period = 12 hours → B = 2π/12 = π/6
- Use cosine with maximum at t=2

Answer: h(t) = 5cos(π(t−2)/6) + 7

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 A sound wave has amplitude 0.3 cm and frequency 440 Hz. Write a model y(t) for the displacement.

Hint: Period = 1/frequency. B = 2π/period = 2π·frequency. Assume no phase shift and midline y=0.

2 A sinusoidal function passes through a max of 10 at x=π/4 and a min of −2. Write the equation using cosine.

Hint: Find A and D from max and min. The maximum of cosine occurs at x=0, so a phase shift of π/4 is needed.

3 Identify the amplitude, period, phase shift, and midline of y = −2cos(πx/3 + π/6) + 4.

Hint: Read off A=−2 (amplitude=2), B=π/3, C=π/6, D=4. Phase shift = −C/B.

4 A city's high temperature has a max of 95°F in July (month 7) and a min of 35°F in January (month 1). Predict the average high in April (month 4).

Hint: Build the model first, then evaluate at t=4.

5 A spring's displacement from rest is 8 cm at t=0 and 0 cm at t=π/4 seconds. Amplitude is 8 cm. Write a model d(t).

Hint: The displacement starts at maximum (8 cm) and decreases, suggesting a cosine model with no phase shift.

Key Vocabulary

Amplitude	|A| — half the distance between the maximum and minimum values of the function.
Period	2π/|B| — the horizontal length of one complete cycle.
Phase Shift	−C/B — the horizontal shift of the graph from the parent sine or cosine function.
Midline	y=D — the horizontal line halfway between the maximum and minimum, about which the graph oscillates.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **For y = 5sin(2x − π/3) + 1, the amplitude is:** Multiple Choice

A 1
B 2
C 5
D π/3

2 **For y = 3cos(πx/4) − 2, the period is:** Multiple Choice

A π/4
B π/2
C 4
D 8

3 **A sinusoidal function has max=14 and min=2. The midline is:** Multiple Choice

A y = 6
B y = 7
C y = 8
D y = 12

4 **For y = A·sin(Bx + C) + D, the phase shift is:** Multiple Choice

A C/B
B −C/B
C B/C
D −B/C

5 **A Ferris wheel with radius 15m and center 18m high completes one revolution in 40 seconds. The amplitude of the height model is:** Multiple Choice

A 18
B 15
C 33
D 3

Common Mistakes to Avoid

✗ Using the maximum value as the amplitude instead of half the range.	✓ Amplitude A = (max−min)/2. The maximum value is D+A, not A itself.
✗ Setting B equal to the period instead of 2π/period.	✓ B = 2π/period. If the period is 12, then B = 2π/12 = π/6.
✗ Confusing phase shift direction: thinking a positive C shifts right.	✓ In y=A·sin(Bx+C)+D, phase shift = −C/B. A positive C shifts the graph LEFT.
✗ Forgetting to account for the midline when reading max/min from a graph.	✓ The maximum of the function is D+A, and the minimum is D−A. Always identify D first.

Math Tips

★ When a problem gives a maximum at a specific x-value, cosine is often the cleaner choice — the maximum of cosine occurs at x=0, so the phase shift equals the x-value of the maximum.

★ When a quantity 'starts at the minimum' (like a Ferris wheel rider at the bottom), use −cos(Bx)+D, which starts at its minimum.

★ The four-step recipe: (1) find A and D from max/min, (2) find B from the period, (3) find C from a known point, (4) write the equation.

★ Always verify your model by checking it gives the correct max and min values, and passes through at least one given data point.

★ Frequency and period are reciprocals: f=1/T and T=1/f. For sound waves, frequency is given in Hz (cycles per second).