Student Workbook

Solving Trigonometric Equations

Name: ________________________________ Date: _________________ Class: _________

ESSENTIAL QUESTION

How do you find all angles that satisfy a trigonometric equation, given that trig functions are periodic and each value is achieved in multiple quadrants?

Lesson Overview

Solving a trig equation means finding all values of the variable that make the equation true. Because trig functions are periodic, there are infinitely many solutions — but they follow a predictable pattern. Strategy: isolate the trig function, find the reference angle, determine which quadrants apply, list all solutions on $[0, 2\pi)$, then add the period for the general solution.

If $\sin\theta=k$: $\theta=\arcsin(k)+2\pi n$ or $\theta=\pi-\arcsin(k)+2\pi n$

Cosine	If $\cos\theta=k$: $\theta = \pm\arccos(k) + 2\pi n$
Tangent	If $\tan\theta=k$: $\theta = \arctan(k) + \pi n$ (period π, not 2π)
Strategy	Isolate → reference angle → quadrants → list $[0,2\pi)$ solutions → add period

Worked Examples

EXAMPLE 1

Solve: $2\sin\theta - 1 = 0$ on $[0, 2\pi)$

- Isolate $\sin\theta$: $\sin\theta = 1/2$. Reference angle: $\arcsin(1/2) = \pi/6$
- $\sin\theta$ is positive in QI and QII
- QI: $\theta = \pi/6$. QII: $\theta = \pi - \pi/6 = 5\pi/6$

Answer: $\theta = \pi/6, 5\pi/6$

EXAMPLE 2

Solve: $\sqrt{3}\tan\theta + 1 = 0$ on $[0, 2\pi)$

- Isolate $\tan\theta$: $\tan\theta = -1/\sqrt{3}$. Reference angle: $\arctan(1/\sqrt{3}) = \pi/6$
- $\tan\theta$ is negative in QII and QIV
- QII: $\theta = \pi - \pi/6 = 5\pi/6$. QIV: $\theta = 2\pi - \pi/6 = 11\pi/6$

Answer: $\theta = 5\pi/6, 11\pi/6$

EXAMPLE 3

Solve: $2\cos^2\theta - \cos\theta - 1 = 0$ on $[0, 2\pi)$

- Factor as a quadratic in $\cos\theta$: $(2\cos\theta + 1)(\cos\theta - 1) = 0$
- Case 1: $\cos\theta = 1 \rightarrow \theta = 0$
- Case 2: $\cos\theta = -1/2 \rightarrow$ ref angle $= \pi/3$; negative in QII, QIII: $\theta = 2\pi/3, 4\pi/3$

Answer: $\theta = 0, 2\pi/3, 4\pi/3$

EXAMPLE 4

Solve: $\sin(2\theta) = \sqrt{3}/2$ on $[0, 2\pi)$

- Let $u = 2\theta$. Since $\theta \in [0, 2\pi)$, $u \in [0, 4\pi)$
- $\sin(u) = \sqrt{3}/2 \rightarrow$ ref angle $= \pi/3$
- $u = \pi/3, 2\pi/3, 7\pi/3, 8\pi/3$. Divide by 2

Answer: $\theta = \pi/6, \pi/3, 7\pi/6, 4\pi/3$

EXAMPLE 5

Solve: $2\sin^2\theta - \sin\theta - 1 = 0$ on $[0, 2\pi)$

- Factor: $(2\sin\theta + 1)(\sin\theta - 1) = 0$
- Case 1: $\sin\theta = 1 \rightarrow \theta = \pi/2$
- Case 2: $\sin\theta = -1/2 \rightarrow$ ref angle$=\pi/6$; negative in QIII, QIV: $\theta=7\pi/6, 11\pi/6$

Answer: $\theta = \pi/2, 7\pi/6, 11\pi/6$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Solve: $2\cos\theta + \sqrt{2} = 0$ on $[0, 2\pi)$

Hint: Isolate $\cos\theta$, find the reference angle, then identify the two quadrants where cosine has that sign.

2 Solve: $\tan^2\theta - 3 = 0$ on $[0, 2\pi)$

Hint: Solve for $\tan\theta$ (two values: $\pm\sqrt{3}$), then find all four solutions.

3 Solve: $2\sin^2\theta + \sin\theta = 0$ on $[0, 2\pi)$

Hint: Factor out $\sin\theta$ first, then solve each factor.

4 Solve: $\cos(2\theta) = 1/2$ on $[0, 2\pi)$

Hint: Let $u = 2\theta$, expand the domain to $[0, 4\pi)$, find all u solutions, then divide by 2.

5 Write the general solution for: $\sin\theta = -\sqrt{3}/2$

Hint: Find the two solutions on $[0, 2\pi)$, then add $2\pi n$ to each.

Key Vocabulary

Reference Angle	The acute angle between the terminal side and the x-axis; used to find all solutions from one known value.
General Solution	A formula (using $+2\pi n$ or $+\pi n$) that captures every solution of a periodic equation, not just those on $[0,2\pi)$.
Extraneous Solution	A value produced by an algebraic step (like squaring) that does not satisfy the original equation — must be checked and discarded.
Quadratic in cosθ (or sinθ)	An equation like $2\cos^2\theta - \cos\theta - 1 = 0$ that can be factored the same way as a quadratic in x.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **The solutions of $\sin\theta = 1/2$ on $[0, 2\pi)$ are:** Multiple Choice

A $\pi/6$ only
B $\pi/6$ and $5\pi/6$
C $\pi/3$ and $2\pi/3$
D $\pi/6$ and $11\pi/6$

2 **To solve $\sin(2\theta) = k$ on $[0, 2\pi)$, the domain for 2θ is:** Multiple Choice

A $[0, \pi)$
B $[0, 2\pi)$
C $[0, 4\pi)$
D $[0, 8\pi)$

3 **The general solution for $\cos\theta = 0$ is:** Multiple Choice

A $\theta = \pi/2 + 2\pi n$
B $\theta = \pi/2 + \pi n$
C $\theta = \pi + 2\pi n$
D $\theta = \pi n$

4 **To solve $2\cos^2\theta - \cos\theta - 1 = 0$, the first step is:** Multiple Choice

- **A** Use the quadratic formula
- **B** Factor as $(2\cos\theta + 1)(\cos\theta - 1) = 0$
- **C** Take the square root of both sides
- **D** Divide by $\cos\theta$

5 **How many solutions does $\sin\theta = -1/2$ have on $[0, 2\pi)$?** Multiple Choice

- **A** 1
- **B** 2
- **C** 3
- **D** 4

Common Mistakes to Avoid

✗ Finding only one solution (the reference angle) and forgetting the second quadrant solution.

✓ Every trig equation with $|k|<1$ has two solutions per period. Always check both quadrants where the sign is correct.

✗ When solving $\sin(2\theta)=k$ on $[0,2\pi)$, only looking for solutions in $[0,2\pi)$ for the argument 2θ.

✓ If $\theta\in[0,2\pi)$, then $2\theta\in[0,4\pi)$. You must find all solutions of $\sin(u)=k$ in $[0,4\pi)$.

✗ Dividing both sides by $\sin\theta$ (or $\cos\theta$) when solving a factored equation, losing solutions where $\sin\theta=0$.

✓ Factor instead of dividing. Dividing by a trig function discards the solutions where that function equals zero.

✗ Writing the general solution for $\tan\theta=k$ as $\theta=\arctan(k)+2\pi n$.

✓ Tangent has period π, not 2π. The general solution is $\theta=\arctan(k)+\pi n$.

Math Tips

★ Always isolate the trig function first before finding the reference angle.

★ Draw a quick unit circle sketch and mark the quadrants where the trig function has the required sign. This prevents missing solutions.

★ For equations with double angles ($\sin(2\theta)$, $\cos(2\theta)$), double the domain before solving, then halve all solutions at the end.

★ Tangent has period π, so its general solution adds πn. Sine and cosine have period 2π, so they add $2\pi n$.

★ After finding solutions, always verify by substituting back into the original equation — especially for quadratic trig equations where extraneous solutions can appear.