

Student Workbook

Inverse Trigonometric Functions

Learning Target: I can evaluate inverse trigonometric functions and compositions of trig and inverse trig functions.

Vocabulary

Term	Meaning
$\arcsin(x)$	Domain $[-1, 1]$, Range $[-\pi/2, \pi/2]$
$\arccos(x)$	Domain $[-1, 1]$, Range $[0, \pi]$
$\arctan(x)$	Domain all reals, Range $(-\pi/2, \pi/2)$

Key Concept: Why We Restrict the Domain

SINE, COSINE, AND TANGENT ARE PERIODIC They fail the horizontal line test over their full domain, so their inverses only exist if we restrict to an interval where they ARE one-to-one. This restricted interval becomes the RANGE of the inverse function.

EXAMPLE 1
Evaluate $\arcsin(1/2)$. We need the angle in $[-\pi/2, \pi/2]$ whose sine is $1/2$.

$\arcsin(1/2) = \rule{1.5cm}{0.4pt}$

EXAMPLE 2
Evaluate $\arccos(-\sqrt{2}/2)$. We need the angle in $[0, \pi]$ whose cosine is $-\sqrt{2}/2$.

$\arccos(-\sqrt{2}/2) = \rule{1.5cm}{0.4pt}$

COMMON MISTAKE
 $\arcsin(\sin(x))$ does NOT always equal x ! It only works when x is already inside $\arcsin$'s range, $[-\pi/2, \pi/2]$.

Key Concept: Composition of Different Trig Functions

EXAMPLE 3

Evaluate $\sin(\arccos(3/5))$. Let $\theta = \arccos(3/5)$, so $\cos\theta = 3/5$ with θ in $[0, \pi]$ (Quadrant I, since cosine is positive).

Using the Pythagorean identity or a 3-4-5 triangle: $\sin\theta = \underline{\hspace{2cm}}$

Guided Practice — Part A: Evaluate arcsin & arccos

A1 Evaluate $\arcsin(\sqrt{3}/2)$.

A2 Evaluate $\arccos(1/2)$.

Guided Practice — Part B: Evaluate arctan

B1 Evaluate $\arctan(\sqrt{3})$.

B2 Evaluate $\arctan(0)$.

Guided Practice — Part C: Same-Function Composition

C1 Evaluate $\cos(\arccos(0.7))$.

C2 Evaluate $\sin(\arcsin(-0.4))$.

Guided Practice — Part D: Different-Function Composition

D1 Evaluate $\cos(\arcsin(3/5))$.

D2 Evaluate $\tan(\arccos(5/13))$.

BEFORE YOU MOVE ON

Check with a partner: for compositions of different functions, did you sketch a right triangle or use the Pythagorean identity to find the missing side?