

Unit 3 Test — Answer Key

End Behavior, Zeros, Asymptotes & Graphing Techniques

FOR TEACHER / SELF-CHECK USE

This test was written to cover Unit 3's topics (end behavior, zeros, asymptotes, and graphing techniques for polynomial and rational functions). Every solution below has been independently verified.

1	Leading term $-2x^4$: even degree with negative leading coefficient, so both ends fall toward $-\infty$. Answer: A — Both ends go to $-\infty$
2	Factor: $x(x^2-4) = x(x-2)(x+2) = 0 \rightarrow x = 0, 2, -2$. Answer: A — 0, 2, -2
3	Synthetic division by 3: coefficients 1, -2, -5, 6. Bring down 1; $1 \times 3 + (-2) = 1$; $1 \times 3 + (-5) = -2$; $-2 \times 3 + 6 = 0$. Answer: Quotient: $x^2 + x - 2$; Remainder: 0
4	Denominator $x^2-9 = 0 \rightarrow x = \pm 3$. Numerator $(x+1)$ is nonzero at both, so both are vertical asymptotes. Answer: A — $x = 3$ and $x = -3$
5	Numerator and denominator have equal degree (2); the horizontal asymptote is the ratio of leading coefficients: $3/1$. Answer: A — $y = 3$
6	Factor the denominator: $x^2-x-6 = (x-3)(x+2)$. Domain excludes $x = 3$ and $x = -2$. Answer: All real numbers except $x = 3$ and $x = -2$
7	Testing $x=2$: $8-12-8+12=0$, so $(x-2)$ is a factor. $x^3-3x^2-4x+12 = (x-2)(x^2-x-6) = (x-2)(x-3)(x+2)$. Answer: A — -2, 2, 3
8	A slant asymptote occurs when the numerator's degree is exactly one more than the denominator's. Only $(x^2+1)/(x-1)$ fits (degree 2 over degree 1). Answer: A — $f(x) = (x^2 + 1)/(x - 1)$
9	Divide x^2+3x+1 by $x-2$: $x^2 \div x = x$; $x(x-2) = x^2-2x$; remainder $5x+1$. $5x \div x = 5$; $5(x-2) = 5x-10$; remainder 11. Quotient: $x+5$, remainder 11. As $x \rightarrow \pm\infty$, the remainder term vanishes, leaving the slant asymptote $y = x + 5$. Answer: $y = x + 5$
10	Factor: $x^2-4 = (x-2)(x+2)$. $f(x) = (x-2)(x+2)/(x-2) = x+2$ for $x \neq 2$. At $x=2$, the simplified function gives $y = 2+2 = 4$, but f is undefined there. Answer: $f(x)$ simplifies to $x+2$ ($x \neq 2$); hole at (2, 4)