Student Workbook

Unit Review — Polynomial, Radical & Rational Functions

Name: ______________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

How do complex numbers, quadratic and polynomial models, rational functions, and variation work together to describe and solve algebraic relationships?

Unit Overview

Unit 3 covered the richest family of algebraic functions. Complex numbers (using $i^2=-1$) let every quadratic have a solution. Vertex form reveals a parabola's maximum or minimum. Polynomial graphs are shaped by degree, leading coefficient (end behavior), and zeros with multiplicity. Synthetic division and the Remainder/Rational Zero Theorems help factor higher-degree polynomials. Rational functions have asymptotes and holes, and direct, inverse, and joint variation model proportional relationships.

$$i^2 = -1$$

$$f(x) = a(x-h)^2 + k \text{ (vertex form)}$$

$$x = (-b \pm \text{sqrt}(b^2-4ac)) / (2a)$$

Zeros	Multiplicity even -> graph touches x-axis; odd -> graph crosses
Rational	Cancel common factors first: canceled factors are HOLES, not asymptotes
Variation	Direct: y=kx. Inverse: y=k/x. Joint: y=kxz

Quick Reference

- Powers of i cycle every 4: i, -1, -i, 1. Divide the exponent by 4 and use the remainder.
- Horizontal asymptote: compare degrees. num<denom -> y=0; equal -> ratio of leading coefficients; num>denom -> none (check oblique).
- Rational Zero Theorem lists POSSIBLE rational zeros (factors of constant / factors of leading coeff) — you must still test them.

Worked Examples

EXAMPLE 1

Simplify (3+4i)(2-5i), and give the conjugate of your answer.

- Expand: $6 - 15i + 8i - 20i^2$
- $i^2 = -1$, so $-20i^2 = 20$: $6 - 7i + 20$

Answer: (3+4i)(2-5i) = 26 - 7i; conjugate = 26 + 7i

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EXAMPLE 2

Write $f(x) = 2x^2 - 8x + 3$ in vertex form and find the vertex.

- Factor out 2: $f(x) = 2(x^2 - 4x) + 3$
- Complete the square inside: $2(x^2-4x+4-4)+3 = 2(x-2)^2 - 8 + 3$

Answer: $f(x) = 2(x-2)^2 - 5$; vertex = (2, -5)

EXAMPLE 3

Find the zeros (with multiplicity) and end behavior of $f(x) = (x-1)^2(x+3)$.

- Zeros: x=1 (multiplicity 2 -> touches x-axis); x=-3 (multiplicity 1 -> crosses)
- Degree 3 (odd), leading coefficient +1

Answer: Falls left, rises right; touches at x=1, crosses at x=-3

EXAMPLE 4

Use synthetic division to divide $x^3 - 4x^2 + x + 6$ by (x-3), and fully factor.

- Coefficients 1, -4, 1, 6 with root 3: bring down 1; 1(3)=3, -4+3=-1; -1(3)=-3, 1-3=-2; -2(3)=-6, 6-6=0
- Quotient: $x^2 - x - 2 = (x-2)(x+1)$, remainder 0 -> f(3)=0

Answer: $f(x) = (x-3)(x-2)(x+1)$

EXAMPLE 5

Find the holes and asymptotes of $f(x) = (x^2-4) / (x^2-x-6)$.

- Factor: (x-2)(x+2) / (x-3)(x+2) -> cancel (x+2): hole at x=-2
- Simplified: (x-2)/(x-3). Vertical asymptote x=3. Degrees equal -> horizontal asymptote y=1
- Hole y-value: (-2-2)/(-2-3) = -4/-5 = 4/5

Answer: VA: x=3; HA: y=1; hole at (-2, 4/5)

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Simplify (5-2i)+(3+7i) and (5-2i)-(3+7i).

Hint: Combine real parts together and imaginary parts together.

2 Solve $3x^2 - 5x - 2 = 0$ using the quadratic formula.

Hint: a=3, b=-5, c=-2. Compute the discriminant first: b^2-4ac.

3 Find all zeros (with multiplicity) of $f(x) = x^4 - 5x^2 + 4$, and describe the end behavior.

Hint: Let $u=x^2$ and factor the quadratic in u first, then factor each factor again.

4 Use synthetic division to divide $2x^3 + 3x^2 - 11x - 6$ by $(x+3)$, and fully factor.

Hint: Use root -3 in synthetic division, then factor the resulting quadratic.

5 Find the holes and asymptotes of $f(x) = (x+1) / (x^2-1)$.

Hint: Factor the denominator as a difference of squares, then cancel common factors.

Key Vocabulary

Complex Number	A number of the form a+bi, where i is the imaginary unit and $i^2=-1$.
Conjugate	The complex conjugate of a+bi is a-bi; used to simplify complex fractions.
Vertex Form	A quadratic written as $f(x)=a(x-h)^2+k$, where (h,k) is the vertex.
Multiplicity	How many times a zero is repeated as a factor; even multiplicity touches, odd crosses the x-axis.
End Behavior	The direction the graph goes as x -> ±∞, determined by degree and leading coefficient.
Remainder Theorem	f(c) equals the remainder when f(x) is divided by (x-c).
Vertical Asymptote	A vertical line x=c that a rational function's graph approaches but never touches (uncanceled zero of the denominator).
Variation	Direct (y=kx), inverse (y=k/x), or joint (y=kxz) proportional relationships.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **Simplify i^{23}.** Multiple Choice

A i
B -i
C 1
D -1

2 **Find the vertex of $f(x) = x^2 + 6x + 5$.** Multiple Choice

- **A** (-3,-4)
- **B** (3,-4)
- **C** (-3,4)
- **D** (6,5)

3 **What is the end behavior of $f(x) = -2x^4 + 3x - 1$?** Multiple Choice

- **A** Rises left, falls right
- **B** Falls both ends
- **C** Rises both ends
- **D** Falls left, rises right

4 **Which is a vertical asymptote of $f(x) = (x+2)/(x^2-4)$?** Multiple Choice

- **A** x = -2
- **B** x = 2
- **C** x = 4
- **D** x = 0

5 **If y varies inversely with x, and y=8 when x=3, find y when x=6.** Multiple Choice

- **A** 16
- **B** 4
- **C** 24
- **D** 2

Common Mistakes to Avoid

✗ Forgetting that $i^2=-1$ and treating i like a normal variable when multiplying complex numbers.

✓ Always simplify i^2 to -1 (and $i^3=-i$, $i^4=1$) after expanding.

✗ Completing the square without factoring out 'a' from the x^2 and x terms first.

✓ Factor out a first, complete the square INSIDE the parentheses, then distribute back out.

✗ Assuming every zero of a rational function's denominator is a vertical asymptote.

✓ Factor and cancel common factors first — canceled factors are holes, not asymptotes.

✗ Assuming every zero of a polynomial crosses the x-axis.

✓ Even multiplicity means the graph touches and turns around; odd multiplicity means it crosses.

Math Tips

- ★ Powers of i cycle every 4: i, -1, -i, 1. Divide the exponent by 4 and use the remainder.
- ★ Quick vertex shortcut: h = -b/(2a), then k = f(h) — no need to complete the square every time.
- ★ Horizontal asymptotes compare degrees of numerator and denominator, not just the coefficients.
- ★ The Rational Zero Theorem only lists POSSIBLE rational zeros — test them with synthetic division.
- ★ For variation problems, find k first from the given values, then reuse that same k for the question.