Unit 12 Test — Answer Key

Limits, Continuity & the Derivative as a Rate of Change

FOR TEACHER / SELF-CHECK USE

This test was written to cover Unit 12's topics (evaluating limits, continuity, and the derivative as an instantaneous rate of change — a preview of calculus). Every solution below has been independently verified.

1 Factor: $(x^2-4)/(x-2) = (x-2)(x+2)/(x-2) = x+2$ for $x \neq 2$. As $x \to 2$, this approaches $2+2$.

Answer: A — 4

2 Factor: $(x^2-9)/(x-3) = (x-3)(x+3)/(x-3) = x+3$ for $x \neq 3$. As $x \to 3$, this approaches $3+3$.

Answer: A — 6

3 Factor: $(x^2-1)/(x-1) = (x-1)(x+1)/(x-1) = x+1$ for $x \neq 1$. As $x \to 1$, this approaches $1+1$.

Answer: 2

4 The function is undefined (division by zero) when $x-3=0$, i.e. $x=3$, which is a discontinuity (vertical asymptote).

Answer: A — x = 3

5 $|x|$ has no gaps, jumps, or holes at $x=0$ (it is continuous), but its graph has a sharp corner there, so it is not differentiable.

Answer: A — It is continuous but not differentiable

6 Average rate of change = $[f(5)-f(2)]/(5-2) = (25-4)/3 = 21/3$.

Answer: 7

7 $f'(x) = \lim(h\to 0)\ [(x+h)^2 - x^2]/h = \lim(h\to 0)\ [2xh+h^2]/h = \lim(h\to 0)\ (2x+h) = 2x$.

Answer: A — f'(x) = 2x

8 Since $f'(x) = 2x$ (from the previous problem), $f'(3) = 2(3)$.

Answer: A — 6

9 When numerator and denominator have the same degree (both degree 2), the limit at infinity equals the ratio of the leading coefficients, since lower-degree terms become negligible.
Ratio of leading coefficients: 3/1.

Answer: 3

10 $f'(x) = \lim(h\to 0)\ [f(x+h) - f(x)]/h$.
$f(x+h) = (x+h)^2 + 3(x+h) = x^2+2xh+h^2+3x+3h$.
$f(x+h) - f(x) = 2xh+h^2+3h$. Divide by h: $2x+h+3$.
As $h\to 0$: $2x+0+3$.

Answer: f'(x) = 2x + 3