Student Workbook

Series and Their Notations

Name: ______________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

How can sigma notation compactly represent the sum of many terms, and what formulas let us find those sums efficiently?

Lesson Overview

A series is the sum of the terms of a sequence. When a sequence has many terms, writing out every addition is tedious. Sigma (Σ) notation gives us a compact way to express any sum. The partial sum S_n is the sum of the first n terms. For arithmetic and geometric sequences, closed-form formulas let us compute S_n without listing every term. For infinite geometric series with $|r| < 1$, the sum converges to a finite value $S_\infty = a_1 / (1 - r)$.

$$S_n = n/2 \cdot (a_1 + a_n)$$
$$S_n = a_1(1 - r^n)/(1 - r)$$
$$S_\infty = a_1/(1 - r),\ |r| < 1$$

Arithmetic Series	$S_n = n/2 \cdot (a_1 + a_n) = n/2 \cdot (2a_1 + (n-1)d)$.
Finite Geometric Series	$S_n = a_1(1 - r^n) / (1 - r)$, for $r \neq 1$.
Infinite Geometric Series	$S_\infty = a_1 / (1 - r)$ when $|r| < 1$; diverges when $|r| \geq 1$.
Sigma Notation	$\Sigma(k=m \text{ to } n)\ a_k$ means add a_k for every integer k from m to n.

Worked Examples

EXAMPLE 1

Evaluate Σ(k=1 to 5) (2k − 1) by listing terms and summing.

- k=1: 2(1)−1=1; k=2: 2(2)−1=3; k=3: 2(3)−1=5; k=4: 2(4)−1=7; k=5: 2(5)−1=9
- Sum all terms: 1 + 3 + 5 + 7 + 9

Answer: 25

EXAMPLE 2

Find S_{10} for the arithmetic series with $a_1 = 3$ and $d = 4$.

- Find a_{10}: $a_{10} = 3 + (10 - 1)(4) = 3 + 36 = 39$
- Apply formula: $S_{10} = 10/2 \cdot (a_1 + a_{10}) = 5 \cdot (3 + 39) = 5 \cdot 42$

Answer: $S_{10} = 210$

EXAMPLE 3

Find S_6 for the geometric series with $a_1 = 2$ and $r = 3$.

- Apply formula: $S_6 = a_1(1 - r^6) / (1 - r)$; $r^6 = 3^6 = 729$
- $S_6 = 2(1 - 729) / (1 - 3) = 2(-728) / (-2) = -1456 / -2$

Answer: $S_6 = 728$

EXAMPLE 4

Find S_∞ for the infinite geometric series with $a_1 = 8$ and $r = 1/2$.

- Check convergence: $|r| = 0.5 < 1$ ✓ — series converges
- $S_\infty = a_1 / (1 - r) = 8 / (1 - 1/2) = 8 / (1/2) = 8 \times 2$

Answer: $S_\infty = 16$

EXAMPLE 5

Write sigma notation for the series 4 + 8 + 12 + 16 + 20 + 24.

- Each term is a multiple of 4: term k is $a_k = 4k$
- First term: k=1 gives 4(1)=4 ✓; Last term: k=6 gives 4(6)=24 ✓

Answer: Σ(k=1 to 6) 4k

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Evaluate Σ(k=1 to 4) (3k + 1) by listing each term then summing.

Hint: Substitute k = 1, 2, 3, 4 into (3k + 1) to get four terms, then add them.

2 Find S_8 for the arithmetic series with $a_1 = 5$ and $d = 3$.

Hint: First find a_8 using $a_n = a_1 + (n-1)d$, then use $S_8 = 8/2 \cdot (a_1 + a_8)$.

3 Find S_5 for the geometric series with $a_1 = 1$ and $r = 4$.

Hint: Use $S_n = a_1(1 - r^n)/(1 - r)$. Compute $4^5 = 1024$ first.

4 Find S_∞ for the infinite geometric series with $a_1 = 6$ and $r = 1/3$.

Hint: Check $|r| < 1$, then apply $S_\infty = a_1/(1 - r)$. Simplify the fraction carefully.

5 Write sigma notation for 1 + 4 + 9 + 16 + 25.

Hint: Notice each term is a perfect square: $1^2, 2^2, 3^2, 4^2, 5^2$. What is the general term a_k?

Key Vocabulary

Series	The sum of the terms of a sequence.
Sigma notation (Σ)	A compact notation using the Greek letter Σ to represent a sum, with an index variable, lower bound, and upper bound.
Partial sum (S_n)	The sum of the first n terms of a series.
Arithmetic series	The sum of an arithmetic sequence; computed with $S_n = n/2\cdot(a_1+a_n)$.
Geometric series	The sum of a geometric sequence; computed with $S_n = a_1(1-r^n)/(1-r)$.
Infinite geometric series	A geometric series with infinitely many terms; converges to $S_\infty = a_1/(1-r)$ when $|r| < 1$.

Quick Check Quiz

Circle the letter of the correct answer.

1 **Which formula gives the sum of an infinite geometric series when |r| < 1?** Multiple Choice

A $S_\infty = a_1 / (1 - r)$

B $S_\infty = a_1 \cdot r^\infty$

C $S_\infty = n/2 \cdot (a_1 + a_n)$

D $S_\infty = a_1(1 - r^n)/(1 - r)$

Common Mistakes to Avoid

✗ Using the infinite series formula $S_\infty = a_1/(1-r)$ when $|r| \geq 1$.

✓ Always check $|r| < 1$ first. If $|r| \geq 1$, the series diverges and has no finite sum.

✗ Forgetting to include the lower bound when evaluating sigma notation (starting at $k = 0$ instead of $k = 1$).

✓ Always read the lower bound carefully — it tells you the first value of k to substitute.

✗ Using the arithmetic formula $S_n = n/2\cdot(a_1+a_n)$ for a geometric series.

✓ Match the formula to the series type: arithmetic uses d, geometric uses r.

✗ Computing r^n as $r\cdot n$ (multiplying instead of raising to a power).

✓ r^n means r multiplied by itself n times: e.g., $3^4 = 81$, not $3\times4 = 12$.

Math Tips

★ For small sigma sums ($n \le 6$), listing every term and adding is often faster and less error-prone than using a formula.

★ The arithmetic series formula $S_n = n/2\cdot(a_1+a_n)$ is the average of the first and last terms, times the number of terms.

★ For infinite geometric series, think of $S_\infty = a_1/(1-r)$ as 'first term divided by (1 minus ratio).'

★ When writing sigma notation, always verify your formula by plugging in the first and last index values.

★ A geometric series with $r = -1/2$ still converges because $|-1/2| = 0.5 < 1$ — only the absolute value of r matters for convergence.