Student Workbook

Geometric Sequences

Name: ______________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

How can we model situations where quantities multiply by a constant factor — and predict any term without listing them all?

Lesson Overview

A geometric sequence is a list of numbers where each term is found by multiplying the previous term by a fixed number called the common ratio r. Geometric sequences model exponential growth and decay: compound interest, population growth, radioactive decay, and bouncing ball heights.

$$r = a_n / a_{n-1}$$
$$a_n = a_1 \cdot r^{n-1}$$
$$a_n = r \cdot a_{n-1}$$

Common Ratio	$r = a_n / a_{n-1}$ — the constant multiplier between consecutive terms; must be constant.
Explicit Formula	$a_n = a_1 \cdot r^{n-1}$ — gives any term directly from its position.
Recursive Formula	$a_n = r \cdot a_{n-1}$ — each term built by multiplying the one before it by r.
Geometric Mean	The value g such that a, g, b is geometric; $g = \sqrt{ab}$.

Worked Examples

EXAMPLE 1

Find the first 5 terms of the geometric sequence with $a_1 = 3$ and $r = 2$.

- $a_1 = 3$
- $a_2 = 3 \cdot 2 = 6$; $a_3 = 6 \cdot 2 = 12$; $a_4 = 12 \cdot 2 = 24$; $a_5 = 24 \cdot 2 = 48$

Answer: 3, 6, 12, 24, 48

EXAMPLE 2

Find a_8 for the geometric sequence with $a_1 = 5$ and $r = -2$.

- Explicit formula: $a_n = a_1 \cdot r^{n-1}$
- $a_8 = 5 \cdot (-2)^{8-1} = 5 \cdot (-2)^7$; $(-2)^7 = -128$
- $a_8 = 5 \cdot (-128) = -640$

Answer: $a_8 = -640$

EXAMPLE 3

Find the common ratio r and the explicit formula given $a_2 = 6$ and $a_5 = 162$.

- From a_2 to a_5 there are 5 − 2 = 3 steps, so $a_5 = a_2 \cdot r^3$
- $162 = 6 \cdot r^3 \rightarrow r^3 = 27 \rightarrow r = 3$
- Find a_1: $a_2 = a_1 \cdot r \rightarrow 6 = a_1 \cdot 3 \rightarrow a_1 = 2$

Answer: $r = 3$; $a_n = 2 \cdot 3^{n-1}$

EXAMPLE 4

Insert 2 geometric means between 4 and 108.

- Sequence: 4, g_1, g_2, 108 — four terms total. $a_4 = a_1 \cdot r^3$
- $108 = 4 \cdot r^3 \rightarrow r^3 = 27 \rightarrow r = 3$
- $g_1 = 4 \cdot 3 = 12$; $g_2 = 12 \cdot 3 = 36$

Answer: The two geometric means are 12 and 36. Sequence: 4, 12, 36, 108.

EXAMPLE 5

$1,000 is invested at 6% annual compound interest. Find the value after 10 years.

- Use $A = P(1 + r)^n$ where P = 1000, r = 0.06, n = 10
- $A = 1000 \cdot (1.06)^{10}$; $(1.06)^{10} \approx 1.7908$
- $A \approx 1000 \cdot 1.7908 = 1790.85$

Answer: ≈ $1,790.85

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Find the first 5 terms of the geometric sequence with $a_1 = 4$ and $r = 3$.

Hint: Multiply each term by 3 to get the next. Start with 4.

2 Find a_6 for the geometric sequence with $a_1 = 2$ and $r = -3$.

Hint: Use $a_n = a_1 \cdot r^{n-1}$. Substitute n = 6 and be careful with the negative sign: $(-3)^5$.

3 Find r and the explicit formula given $a_1 = 5$ and $a_4 = 40$.

Hint: Use $a_4 = a_1 \cdot r^3$. Solve for r^3 first, then take the cube root.

4 Insert 1 geometric mean between 3 and 75.

Hint: You need 3, g, 75. That's 3 terms, so $a_3 = a_1 \cdot r^2$. Solve for r, then find $g = a_1 \cdot r$.

5 A town has a population of 10,000 and grows 5% per year. Find the population after 6 years.

Hint: Geometric with $a_1 = 10{,}000$, $r = 1.05$. Use $a_n = a_1 \cdot r^{n-1}$ with $n = 7$ (after 6 years is the 7th term).

Key Vocabulary

Geometric sequence	A sequence where each term is found by multiplying the previous term by a constant ratio r.
Common ratio (r)	The constant multiplier between consecutive terms: $r = a_n / a_{n-1}$.
Explicit formula	A formula that gives any term directly: $a_n = a_1 \cdot r^{n-1}$.
Recursive formula	A formula that defines each term using the previous one: $a_n = r \cdot a_{n-1}$.
Geometric mean	The value g such that a, g, b is geometric; $g = \sqrt{ab}$.
Compound interest	Interest calculated on both principal and accumulated interest: $A = P(1 + r)^n$.

Quick Check Quiz

Circle the letter of the correct answer.

1 **Which formula correctly gives the nth term of a geometric sequence?** Multiple Choice

A $a_n = a_1 + (n - 1)d$

B $a_n = a_1 \cdot r^{n-1}$

C $a_n = a_1 \cdot r^n$

D $a_n = r \cdot a_1^{n-1}$

Common Mistakes to Avoid

✗ Using $a_n = a_1 \cdot r^n$ instead of $a_n = a_1 \cdot r^{n-1}$ (off-by-one error).

✓ The exponent is n − 1 because the first term has $r^0 = 1$ (no multiplication yet).

✗ Forgetting the negative sign when r is negative: $(-2)^7 = 128$.

✓ Odd powers of a negative number are negative: $(-2)^7 = -128$.

✗ Confusing arithmetic (add d) with geometric (multiply r) sequences.

✓ Check: if ratios are constant → geometric. If differences are constant → arithmetic.

✗ When inserting k geometric means, using r^2 instead of r^{k+1}.

✓ Inserting k means creates k + 2 total terms. Use $a_{k+2} = a_1 \cdot r^{k+1}$ to find r.

Math Tips

★ To check if a sequence is geometric, divide consecutive terms — the ratio must be the same every time.

★ If $r > 1$, the sequence grows. If $0 < r < 1$, it decays. If $r < 0$, terms alternate in sign.

★ The geometric mean of two numbers a and b is $\sqrt{(ab)}$ — the single term that fits between them geometrically.

★ Compound interest $A = P(1 + r)^n$ is just the explicit formula $a_n = a_1 \cdot r^n$ with $a_1 = P$ and ratio = (1 + r).

★ When $r = 1$, every term equals a_1 — a constant sequence (technically geometric but trivial).