Final Exam — Answer Key

Cumulative Review: Functions through Sequences, Series & Probability

FOR TEACHER / SELF-CHECK USE

This final exam was written to cumulatively cover Units 1 through 11. Every solution below has been independently verified.

1 Swap x and y: $x = 4y - 1$. Solve for y: $x+1 = 4y \rightarrow y = (x+1)/4$.
Answer: A — $f^{-1}(x) = (x + 1)/4$

2 Slope = $(5-(-3))/(2-0) = 8/2 = 4$. y-intercept is −3 (given point (0,−3)).
Answer: A — $y = 4x - 3$

3 Factor: $(x-3)(x+2) = 0 \rightarrow x = 3, -2$.
Answer: A — −2, 3

4 Factor the denominator: $x^2-4 = (x-2)(x+2)$. The numerator $(x+2)$ cancels with the $(x+2)$ factor, leaving a removable discontinuity (hole) at $x=-2$. Only $x=2$ remains as a true vertical asymptote.
Answer: $x = 2$ ($x = -2$ is a hole, not an asymptote)

5 $81 = 3^4$, so $3^x = 3^4$ means $x = 4$.
Answer: A — 4

6 Product rule then power rule: $\log(xy^2) = \log(x) + \log(y^2) = \log(x) + 2\log(y)$.
Answer: $\log(x) + 2\log(y)$

7 This is a standard exact value from the 30-60-90 triangle / unit circle.
Answer: A — $\sqrt{3}/2$

8 Period = $2\pi/b$, where $b = 3$. Period = $2\pi/3$.
Answer: A — $2\pi/3$

9 By the Pythagorean identity, $\sin^2(x) + \cos^2(x) = 1$, so $1 - \sin^2(x) = \cos^2(x)$.
Answer: A — $\cos^2(x)$

10 $2\cos(x) = \sqrt{3} \rightarrow \cos(x) = \sqrt{3}/2$. On $[0,2\pi)$, cosine equals $\sqrt{3}/2$ at reference angle $\pi/6$, in Quadrants I and IV: $x = \pi/6$, $2\pi - \pi/6 = 11\pi/6$.
Answer: $x = \pi/6, 11\pi/6$

11 Magnitude = $\sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169}$.
Answer: A — 13

12 Add the equations: $(x+y)+(x-y) = 10+2 \rightarrow 2x = 12 \rightarrow x = 6$. $y = 10 - 6 = 4$.
Answer: A — (6, 4)

13 Determinant of [[a,b],[c,d]] is $ad-bc$: $2(4) - 3(1) = 8 - 3$.
Answer: 5

14 Divide both sides by 36: $x^2/4 + y^2/9 = 1$. Both terms are positive and added with different denominators — an ellipse.
Answer: A — Ellipse

15

Group and complete the square: $(x^2+2x) + (y^2-4y) = 4$.
$(x^2+2x+1) + (y^2-4y+4) = 4 + 1 + 4 \rightarrow (x+1)^2 + (y-2)^2 = 9$.
Answer: $(x + 1)^2 + (y - 2)^2 = 9$; center $(-1, 2)$, radius 3

16

$a_n = a_1 \cdot r^{n-1}$. $a_6 = 2 \cdot 3^5 = 2 \cdot 243 = 486$.
$S_n = a_1(r^n-1)/(r-1)$. $S_5 = 2(3^5-1)/(3-1) = 2(243-1)/2 = 2(242)/2 = 242$.
Answer: 6th term = 486; sum of first 5 terms = 242